

Problems for chapter 11.

Revision: $F = -k(x - x_0)$ or $a \propto -(x - x_0)$

$$f = \frac{1}{T} \quad \text{and} \quad \omega^2 = \frac{k}{m} \quad \text{with} \quad \omega = 2\pi f$$

$$x - x_0 = A \cos \omega t$$

$$v = \omega A \sin \omega t$$

$$a = -\omega^2 A \cos \omega t$$

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x - x_0)^2 = \frac{1}{2}kA^2$$

$$c = \lambda f \quad k = \frac{2P}{\lambda}$$

$$y = A \sin(\omega t - kx) \quad \text{wave travelling to the right.}$$

- 1.* The free end of a spring is pulled so that the following forces are recorded at the given positions: $\{(F, x) : (120 \text{ N}, 1.20 \text{ m}), (250 \text{ N}, 1.85 \text{ m}), (380 \text{ N}, 2.50 \text{ m})\}$. Find the spring constant k for this spring and also find x_0 the position of the end of the spring when it is not under load.
- 2.* Use the information from question 1. to determine the acceleration of a mass of 0.60 kg fastened to the free end of the spring when it is at 1.50 m. What is the frequency of the oscillation of this mass? You should give your answer in terms of the unit hertz or s^{-1} .
3. The free end of a spring is pulled so that the following forces are recorded at the given positions: $\{(F, x) : (50 \text{ N}, 0.80 \text{ m}), (65 \text{ N}, 0.90 \text{ m}), (95 \text{ N}, 1.10 \text{ m})\}$. Find the spring constant k for this spring and also find x_0 the position of the end of the spring when it is not under load.
4. Use the information from question 3. to determine the acceleration of a mass of 0.20 kg fastened to the free end of the spring when it is at 1.00 m. What is the frequency of the oscillation of this mass?
- 5.* If the amplitude of the motion of the mass in question 2. is 0.050 m, find general expressions (use those given in the Revision and put in numerical values) for the position of the mass, the speed of the mass and the acceleration of the mass. What are the maximum values of the distance from x_0 , the speed and the acceleration?
6. If the amplitude of the motion of the mass in question 4. is 0.050 m, find general expressions for the position of the mass, the speed of the mass and the acceleration of the mass. What are the maximum values of the distance from x_0 , the speed and the acceleration?
7. By using the symbolic formula or your numerical formula, show for any particular time that $\frac{1}{2}mv^2 + \frac{1}{2}k(x - x_0)^2 = \frac{1}{2}kA^2$.

8.* A transverse wave follows the formula $y = A \sin(\omega t - kx + \phi)$. For this wave the amplitude is 5.0 m, the speed of the wave is 10.0 m s^{-1} and the period $T = 0.0625 \text{ s}$, while the phase constant is 45° . Change the phase constant to radians and find the transverse displacement of this wave at $t = 1.0 \text{ s}$ and $x = 0.25 \text{ m}$.

9. A transverse wave follows the formula $y = A \sin(\omega t - kx + \phi)$. For this wave the amplitude is 0.50 m, the speed of the wave is 8.0 m s^{-1} and the period $T = 0.050 \text{ s}$, while the phase constant is $\frac{\pi}{2}$. Find the transverse displacement of this wave at $t = 1.0 \text{ s}$ and $x_0 = 0.25 \text{ m}$.

10.* A longitudinal wave has the form $x - x_0 = A \sin(\omega t - kx_0)$, in this expression the phase constant is zero. Use the values of $A = 5.0 \text{ m}$, $\omega = 100 \text{ rad s}^{-1}$ and $k = 10.0 \text{ rad m}^{-1}$, to find the longitudinal displacement of this wave at $t = 2.0 \text{ s}$ and $x_0 = 0.50 \text{ m}$.

11. A longitudinal wave has the form $x - x_0 = A \sin(\omega t - kx_0)$, in this expression the phase constant is zero. Use the values of $A = 0.20 \text{ m}$, $T = 0.25 \text{ s}$ and $\lambda = 1.25 \text{ m}$, to find the longitudinal displacement of this wave at $t = 1.0 \text{ s}$ and $x_0 = 0.50 \text{ m}$.

12.* A travelling wave has a frequency of 0.100 Hz so that $\omega = 2\pi \cdot 0.1 = 0.628 \text{ rad s}^{-1}$ and a wavelength of 2.00 m so that $k = \frac{2\pi}{\lambda} = 3.14 \text{ rad m}^{-1}$. Put the time to zero ($t = 0$) and plot the particular form of $y = 5.0 \sin(\omega t - kx)$ m, for this travelling wave. Use 0.2 m intervals along the x -axis. Next set $t = 1.0 \text{ s}$ and re-plot the wave, can you confirm that it is moving in the positive x -direction?

13. A travelling wave has a frequency of 0.200 Hz and a wavelength of 1.50 m. Put the time to zero ($t = 0$) and plot the particular form of $y = 1.0 \sin(\omega t + kx)$ m, for this travelling wave. Next set $t = 1.0 \text{ s}$ and re-plot the wave, can you confirm that it is moving in the negative x -direction?

14.* A standing transverse wave in a string follows the equation

$$y = 0.002 \sin(2000t) \cos(6.00x) \text{ m.}$$

What is the speed of either of the two travelling waves that form this standing wave? What is the frequency of this standing wave? Locate any two adjacent displacement nodes and also find the wavelength.

15. A standing transverse wave in a string follows the equation

$$y = 0.002 \sin(500t) \cos(3.00x) \text{ m} .$$

What is the speed of either of the two travelling waves that form this standing wave? What is the frequency of this standing wave? Locate any two adjacent displacement nodes and hence or otherwise, find the wavelength.

16.* A 'cello string is 0.60 m long and tuned so that its fundamental frequency is 220 Hz (this is the note "A"), the first harmonic of this string is also the note "A" an octave higher. The second harmonic and the fourth harmonics give the notes that form a major chord (these are the *natural*, *fifth* and *major third* respectively but they are in higher octaves). What are the frequencies of these other two notes?

17. A violin string is 0.30 m long and tuned so that its fundamental frequency is 440 Hz (this is the note "A"), the first harmonic of this string is also the note "A" an octave higher. The second harmonic and the fourth harmonics give the notes that form a major chord (these are the *natural*, *fifth* and *major third* respectively but they are in higher octaves). What are the frequencies of these other two notes? What is the speed of the fundamental transverse waves in the string?

18.* An organ pipe is closed at one end and open at the other so that it can support a transverse acoustic wave. The speed of sound in air is 350 m s^{-1} (this can vary with air temperature and so can the pitch of a pipe organ), if we want our pipe to have a fundamental frequency of 440 Hz we can adjust the length of the pipe so that it is one quarter of a wave length, how long should this pipe be? What are the frequencies of the first, second and third harmonics?

19. An acoustic pipe is open at both ends so that it can support a transverse acoustic wave. The speed of sound in air is 345 m s^{-1} . If we want our pipe to have a fundamental frequency of 440 Hz, how long should the pipe be? What are the frequencies of the first, second, third and fourth harmonics?