CHAPTER 12.

THE PROPAGATION OF LIGHT

12.1 The Nature of Light

Light travels as combined transverse electric and magnetic fields. We have already learned that the electric and magnetic fields are convenient schemes to explain electric and magnetic forces but beyond these measurable properties fields have no other intuitive reality. Light can travel through vacuum, in which case there is no medium for it to interact with. To add to the mystery, Einstein postulated from experimental evidence that light must travel with the nature of discrete particles known as *photons*. Ultimately these explanations of light remain beyond our common intuition. We are left with two theories that adequately explain the nature of light: the first is the Classical Theory using Maxwell's master equations to describe the wave behaviour, the second is Quantum Theory which is able to include the particle nature.

Light travels from source to detector in such a way that it takes the least possible time to travel, this general principle is called Fermat's principle. For all our common considerations we can assume that light travels from source to detector in a straight line. The speed of light is greatest through a vacuum and the value of this speed is

$$c = 3.00\times10^{8}\ \text{m s}^{-1}.$$

Einstein's theory of Special Relativity includes the axiom that nothing can accelerate to speed greater than this value. In all transparent media the speed of light is less than c. The refractive index of a medium is the ratio of c to the speed of light in that medium. The symbol for this refractive index is n (sometimes μ).

$$n_{\text{j}} = \frac{c}{v_{\text{j}}} \qquad (12.1)$$

where v_{j} is the speed of light in a medium that has an index n_{j}. The energy of a photon is proportional to the associated frequency f (or ω) and this frequency does not change as photons cross the boundaries between different media. As a consequence both the wavelength and speed of the photon must change for different media so that

$$f = \frac{v_{\text{j}}}{\lambda_{\text{j}}} = \text{a constant}\ .$$

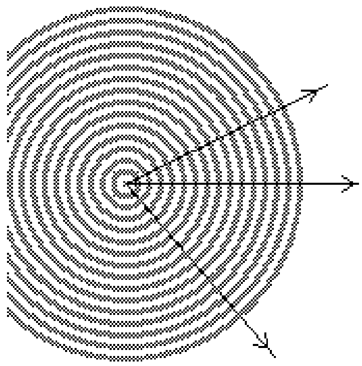
When light encounters a boundary between two transparent media we need not think that the speed changes instantaneously the change occurs over a short distance as the light penetrates the new medium, this distance is called the "skin depth".

The colour of light is technically defined by the wavelength. The human eye rarely encounters *monochromatic* light of a single wavelength or colour. There are no standard wavelength ranges for standard colours; the visible spectrum begins with violet at about 380 nm and ends with red at about 700 nm.

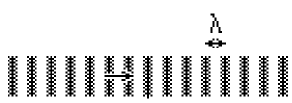
The details of the model for light passing through a refracting medium are quite complicated yet it is commonly found that the refractive index of a medium changes slightly with wavelength, this means that photons will travel at different speeds according to their wavelengths; in particular circumstances the different wavelengths can be separated, this effect is called *dispersion.*

12.2 Rays

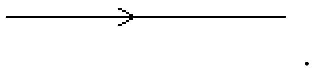
When light travels from a point source it radiates in all directions moving outwards as a spherical set of waves. Most sources give a *spectrum* of wavelengths but for convenience and simplicity we will assume that we are dealing with monochromatic radiation.



In our diagram we have shown a radiating source sending out spherical waves, the waves have all been transmitted at a regular frequency and are evenly spaced as they travel, this regular synchronous behaviour can be called *coherence*. Two sources are coherent when they transmit into the same medium at the same continuous frequency; this does not mean that they must have the same phase, rather that they continue to radiate with the same phase difference. In our diagram we have also included some lines that are perpendicular to the wave-fronts, these lines are called *rays*, they show the direction that a wave must travel (until it meets an obstacle) or the path of an individual photon. (In practise we cannot narrow a light beam down to extremely narrows rays because of *diffraction*.) When the waves have travelled a long way from the source the circular curvature of their wave-fronts reduces so that these wavefronts become parallel. In a very narrow ray the wave fronts are reduced to small sections that can be considered to be parallel. Flat parallel wave fronts in three dimensions are called *plane-waves.* If we could observe the wave-fronts in an optical ray of thickness say 10^{-6} m (also called a *micron*) we would see

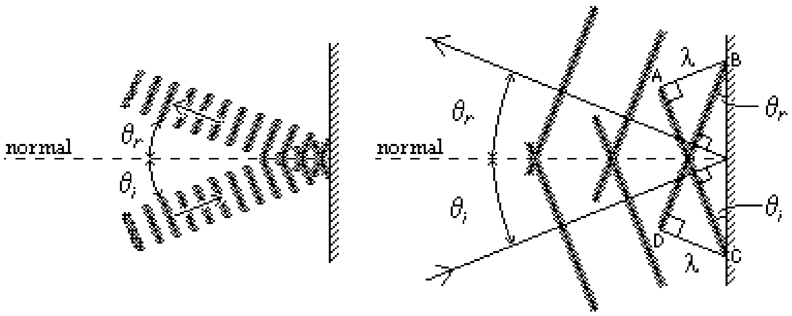


Which we might other wise draw on larger scale as



12.3 Reflection

When a ray meets a medium boundary it can be: absorbed in the new medium, continue through the new medium with a new speed and wavelength or reflect from the boundary. In this section we will consider reflection. Consider (in expanded detail) a ray that meets a plane (flat) boundary between two media. The *angle of incidence* θ_i is the angle between the ray and a perpendicular line from the surface called the *normal.*

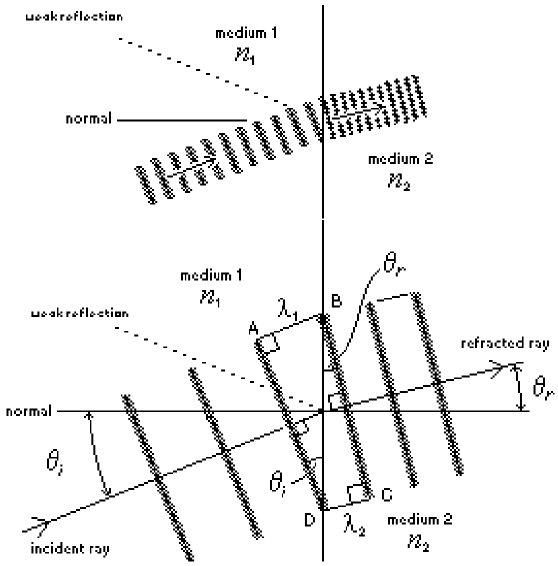


In the above diagram we have shown an expanded ray meeting a fully reflecting surface, this is indicated by the hatching on the surface, a common way for a mirror surface to be indicated. We have then further magnified the diagram so that the (reduced) wavefronts at the surface can form the triangles ABC and DCB with the surface. In these triangles the common sides AB and DC are one wavelength long and the triangles are congruent (with two matching sides and a right angle). This geometry establishes a law of reflection: the incident and reflected rays lie in a plane and the angle of incidence equals the angle of reflection.

$$q_r = q_i$$

12.4 Refraction

When a ray meets a medium boundary it can also continue through the new medium with a new speed and wavelength. In this section we will consider refraction which occurs when a ray enters a medium and is not absorbed but continues through the new medium. Some common examples of refracting media are water or glass. Whenever refraction occurs there is also some reflection, the weakly reflected rays obey the law of reflection, but here we will only concern ourselves with the refracted ray.



This time we have again shown two views of a magnified ray crossing a boundary from medium 1 characterised by a refractive index of n_1, into a medium characterised by n_2. As the waves change speed and wavelength as they encounter the new medium, we have

$$\lambda = \frac{v}{f}$$

where f the frequency remains constant. From the geometry in our last figure

$$\sin \theta_i = \frac{\lambda_1}{\text{BD}}$$

and

$$\sin \theta_r = \frac{\lambda_2}{\text{BD}}$$

so that

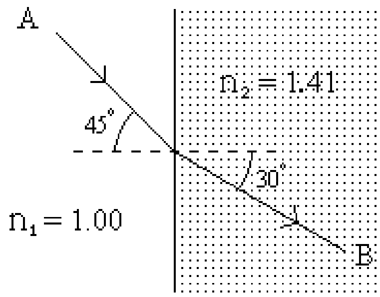
$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

where we have also used equation (12.1). Sometimes the ratio of the refractive indices is also referred to as a refractive index in which case we have

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_2}{n_1} = n_{21} \qquad (12.2)$$

This equation for the ratio of the sines is called Snell's law. The laws of refraction state that: an incident ray and an its refracted component lie in common plane with the surface normal, but on different sides of the surface and the angles of incidence and refraction are related by Snell's law.

When a ray travels from A in medium 1, to B in medium 2, then if the ray travels according to Snell's law the ray takes the least time possible to travel between A and B, we might also call this the shortest path in terms of the time taken.

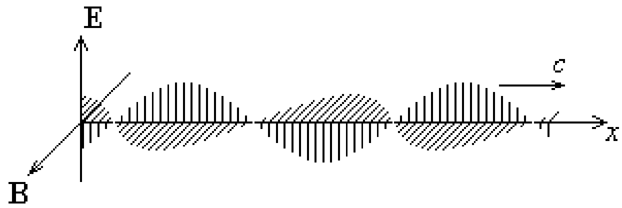


For the particular ray path shown above, it is easy to show that

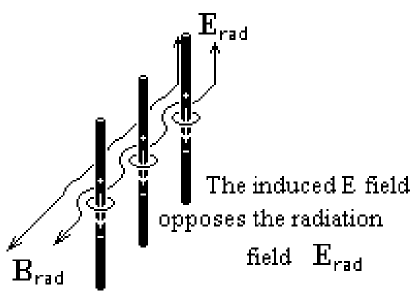
$$\frac{\sin 45°}{\sin 30°} = \frac{1.41}{1.00}$$

12.5 Polarisation

We started this chapter with the statement, "Light travels through a medium as combined transverse electric and magnetic fields.", this transverse property of optical rays helps us understand the nature of polarisation. As we are dealing with electric and magnetic fields we should be prepared to use a little imagination to help our understanding. Consider a monochromatic continuous plane wave moving through space



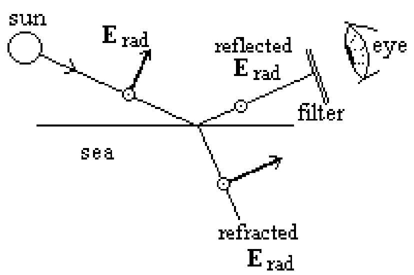
In this sketch the $\mathbf{B}_{rad}$ fields are oscillating horizontally as they move through space, while the $\mathbf{E}_{rad}$ fields oscillate vertically. If these fields encounter a grid of vertical conducting strips $\mathbf{B}_{rad}$ will bend about these conductors and a back e.m.f. will be generated. This will in turn cancel the $\mathbf{E}_{rad}$ field and the wave will not proceed easily through the grid (that is, it is attenuated). The reason for this is suggested by the next diagram, the radiation field $\mathbf{B}_{rad}$ cutting the conducting grid displaces charges in the conductor in such a way that the resulting induced **E** field (shown as white on black) opposes the radiation $\mathbf{E}_{rad}$ field.

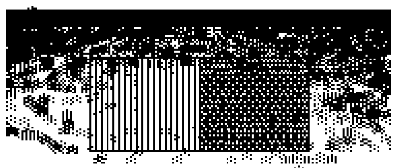


In a polaroid filter the conducting grid is formed by molecular chains and these act to oppose the electric field as described above.

If the polaroid filter is turned sideways, so that the radiation field $\mathbf{B}_{rad}$ is parallel to the conducting grid then little if any opposing field is generated and the radiation passes through without attenuation. Unpolarised light has photons of all orientations of $\mathbf{E}_{rad}$ and $\mathbf{B}_{rad}$ in a direction transverse to the wave motion. Passing this unpolarised light through one filter will then give us (linearly) polarised light where the transverse fields that pass through the filter, have a common alignment. A polarised beam can be analysed by a second filter, at a parallel orientation of the analysing filter the light will not be attenuated at all, but when this filter is turned through a right angle about the propagation axis the beam is wholly attenuated as polarising in both directions are removed by the two filters.

When a beam of light is reflected from smooth a surface the reflected beam can be wholly or partly polarised. The $\mathbf{E}_{rad}$ vector is strongest when it moves across the face of the reflecting surface while the $\mathbf{E}_{rad}$ component that moves into or out of the surface is reduced or removed on reflection. This is the reason while polaroid spectacles can be used effectively to block much of the radiation that is reflected from a smooth water surface or the highway on a sunny day.

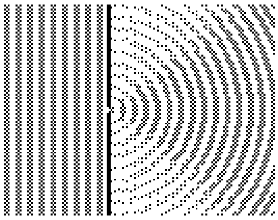




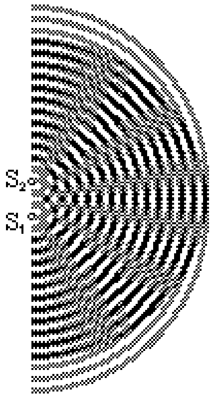
The use of Polariod filters is shown above. The filter with the horizontal molecular chains blocks all the radiation that still reflects from the water, the filter with the vertical molecular chains blocks only a small amount of the reflected radiation.

12.6 Interference

Interference occurs when waves from two or more sources are added together at particular times and places. If the waves have the same amplitudes and are in phase they will combine to give twice the field (or displacement if pressure waves). If the waves are out of phase they subtract to give a zero amplitude and wave intensity. For the purposes of our discussion we need to be more particular and assume that we have two or more monochromatic sources, thus with identical wavelengths, sending out continuous unbroken signals. One way of setting up a suitable system is to have a screen which blocks plane waves with wave crests parallel to the screen but has small openings (apertures) through which some radiation can pass. A single opening in such a screen is shown below, we could be dealing with water surface waves or with spherical light waves that have travelled so far that they have become flat planes.

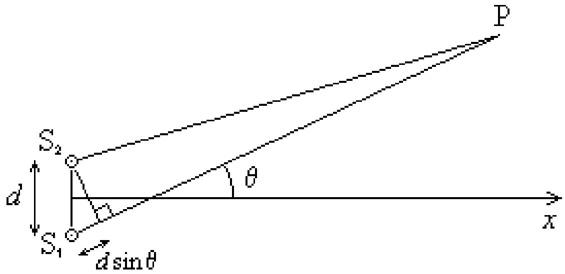


If we have two small sources close together, but more than λ apart, we get the superimposed pattern that is shown next.



This is an interference pattern, the diagram has been arranged so that: two grey wave crests give black and two white troughs stay white, when they are in phase. The waves radiating in the black and white zones have a maximum amplitude and intensity. When the waves are out of phase, the black and white crests and troughs, overlay to give grey, in these zones the waves have little, if any, amplitude and so there is little radiation. The two sources or apertures have been shown as circles (S_1 and S_2) that are a distance *d* apart.

If we go to a *field point* P in a direction θ to the *x*-axis (which is the perpendicular bisector of S_1S_2, we could also call this the *central normal*) then we can calculate the approximate difference in path lengths S_1P and S_2P, this is shown in the diagram below:



From the geometry of this diagram we can see that S_1P is about $d\sin\theta$ longer than S_2P. If two waves start from S_1 and S_2 at the same time, then the wave from S_2 arrives at P first, while the wave from S_1 arrives a little later. We can say that the wave from S_1 is delayed in phase. This phase delay (or phase difference) is the fraction of, the path difference divided by the wavelength, or $\frac{d\sin\theta}{\lambda}$ of a whole cycle. As one wavelength or a whole phase interval is 2π or we can say the phase difference is between the waves from S_1 and S_2 is

$$\Phi = \frac{2\pi d\sin\theta}{\lambda}$$

at P. For our interference conditions the two waves will combine to give a maximum intensity if they arrive at P with the same phase, that is

or $$d\sin\theta = m\lambda \qquad (12.3)$$

where m is an integer. This equation (12.3) is a powerful equation for calculating interference effects, it gives the angle (from the central normal to the two monochromatic coherent sources) at which a maximum intensity will be observed. The minimum (or zero) intensity will occur when the two waves arrive at P out of phase, that is their phases differ by a whole number of cycles plus a half cycle or

$$\Phi = (m + \tfrac{1}{2})2\pi = \frac{2\pi d\sin\theta}{\lambda}$$

or $$d\sin\theta = (m + \tfrac{1}{2})\lambda \qquad (12.4)$$

This is the corresponding equation that enables us to calculate the angle θ, or position at P, at which a minimum or zero intensity is observed in an interference pattern.