CHAPTER 10.

MAGNETIC INDUCTION

10.1 Magnetic Induction

We saw in the last section that an electric current in a magnetic field will experience a force and possible motion (unless the field and current are parallel). A converse effect also occurs; if part of a closed circuit is forced through a magnetic field then a current is generated. This is shown schematically in figure (a) below. Part of a conducting circuit is pulled through a magnetic field **B** by an applied force **F**, as a consequence of this action a current I is generated in the circuit. The direction of the current is such that the right handed field of the current is consistent with the direction of the field lines. These field lines might be thought to bend a little as they are cut by the conductor.

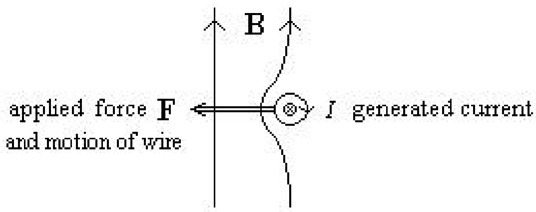


figure (a)

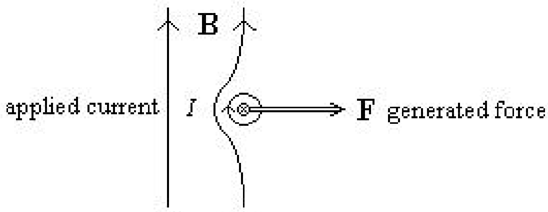


figure (b)

Figure (b) has been taken from the last section, it shows the direction of the force that is generated when a current I flows through the conductor in the magnetic field. *The current generated in figure (a) flows in the direction that creates a force that opposes the force that has generated the current.* When a current is generated it flows to oppose the motion that is causing the current.

You may be tempted to ask, If $F = BIl$, then is the generated current

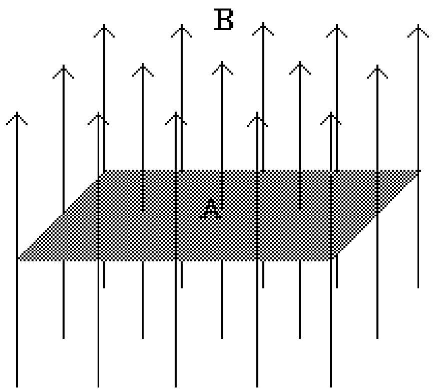
$$I = \frac{F}{Bl} \qquad ?$$

While the generated current gives a reverse force, the equations do not reverse so easily. If the length of circuit that passes through the field is a non-magnetic insulator then no current will be generated and no force is required. If this length of circuit is a perfect conductor we might expect a very large current to be generated by the induction force, again this is not the case, superconductors do exist but they simply will not allow penetration by magnetic field lines. It is safest to say that while our schematic considerations reverse, we cannot so easily reverse the formulae.

10.2 Magnetic flux

The flux of a field is the product of the field strength times the cross sectional area of the field sample that is under consideration. For the diagram below we have the flux Φ as

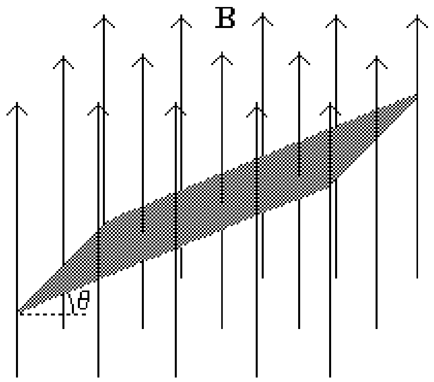
$$\Phi = BA \tag{10.1a}$$



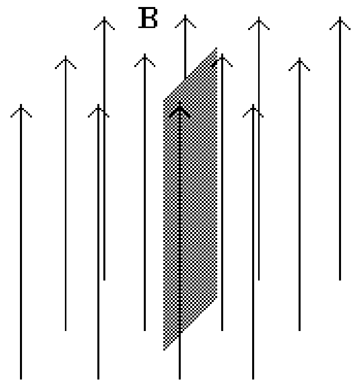
where A is the shaded area through which the field B passes. If the field and the area are not perpendicular we have

$$\Phi = BA\cos\theta \tag{10.1b}$$

where the angle θ is shown in the next diagram.



If θ is 90° the field lines do not cut the area and the flux is zero. This case is shown in the next figure.



10.3 The law of induction

The law for magnetic induction states that the potential energy per charge or *voltage produced around a circuit is proportional to the rate of change of magnetic flux in the circuit*.

$$V = \frac{B_1 A_1 - B_0 A_0}{t_1 - t_0} \tag{10.2}$$

Both the field B and the area of flux A can change, or the field can change with a constant area

$$V = \frac{B_1 - B_0}{t_1 - t_0} A, \tag{10.3}$$

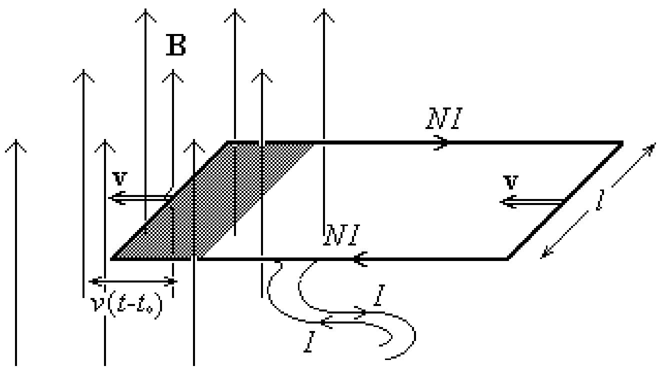
or the area can change with a constant field

$$V = \frac{A_1 - A_0}{t_1 - t_0} B. \tag{10.4}$$

This law often includes a statement to the effect that the current that may flow because of the induced voltage (or e. m. f.), does so to oppose the flux change that is inducing (or generating) the voltage. This effect was seen in section 10.1.

10.4 Induction with Relative Motion

We have already learned that when part of a conducting circuit is pulled through a magnetic field **B** a current I is generated in the circuit. This situation is shown on the next page.



In this diagram a rectangular loop of N turns is moving into a vertical magnetic field **B** with a horizontal velocity **v** (this diagram would also apply to a stationary loop with a field **B** moving across the coil). As result of this motion a current I flows into and out of the loop. You should be able to check that the direction of the induced current is correct, using the right hand rule. From the dimensions shown in the diagram, the area of the field that forms the flux in the coil is $lv(t\text{-}t_0)$, thus we have

$$A - A_0 = lv(t - t_0)$$

and using equation (10.4) we have

$$V = B\frac{lv(t - t_0)}{t - t_0} = Blv \qquad (10.5A)$$

for the voltage induced in a single turn of the coil. The flux must take into account the N turns of the coil so that a voltage of

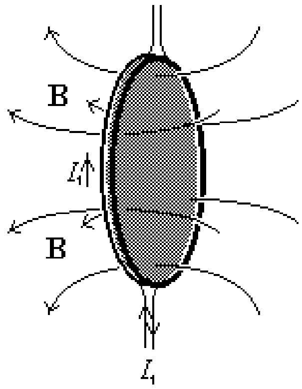
$$V = NB\frac{lv(t - t_0)}{t - t_0} = NBlv \qquad (10.5B)$$

is found between the output leads of the complete coil or circuit.

This discussion can be extended to show that the force on a charge that is free to move around the loop is qvB which is the expected magnetic force formula. This extension is beyond the scope of this course.

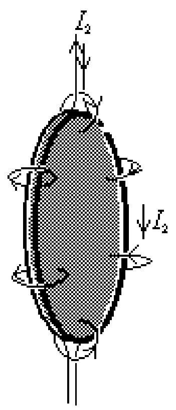
10.5 Induction

Magnetic induction is an important topic, that in itself justifies a study of magnetism. Consider two similar circular loops or coils placed together face to face (shown on the next page).



One coil has N_1 turns and a current I_1 flowing through it, this causes a field **B** that weaves through both coils, both coils have the same flux *BA* as they have a common area *A*. For a constant current I_1 there is no change of flux and thus no current is induced in the second coil.

When the current I_1 changes, the flux changes and a current I_2 is induced in the second coil of N_2 turns. Which direction does this current flow? If I_1 is turned off, the field **B** reduces to zero. In this case we can argue that the field goes to zero by moving out of the area of the coils (speeding away from the coil at the speed of light). We can then suppose that these field lines wrap about the coils as they leave and the right hand rule will give us the direction of the induced current.



The induced current I_2 flows *in the same direction as the previous current I_1 that has been turned off*, we might suppose that it acts to prevent this removal of current. If the current I_1 is turned on then we might imagine that the field lines return and cut through the coils. In this case the induced current I_2 *will flow to oppose the current that has been turned on*. We can also describe this situation by saying that the induced current is opposed to the formation of a fresh current. In general the induced current always acts to oppose whatever change of current or magnetic flux that is inducing the current.

Now suppose that there is a particular change of magnetic flux $\Phi - \Phi_0$ in the system; then over the corresponding time interval $t - t_0$, the N_1 turns of coil 1 will pick up a total voltage of

$$V_1 = N_1 \frac{\mathbf{F} - \mathbf{F}_0}{t - t_0}$$

while the N_2 turns of coil 2 will pick up a total voltage of

$$V_2 = N_2 \frac{\Phi - \Phi_0}{t - t_0} \; .$$

If a voltage V_1 was applied to coil 1 a corresponding current produces a magnetic flux through both coils. When this voltage is turned off the flux disappears, this removal of flux happens rapidly, but not instantly and during this time an average voltage of V_2 appears across the leads of coil 2. If V_1 alternates (that is, turns on and off) it will produce an opposing alternate voltage in coil 2. The relation between the averages values of these voltages can be found from

$$\frac{V_1}{N_1} = \frac{\mathbf{F} - \mathbf{F}_0}{t - t_0} = \frac{V_2}{N_2}$$

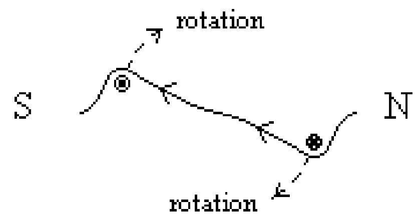
or

$$V_2 = \frac{N_2}{N_1} V_1 \qquad (10.6)$$

This system of two inter-wound coils can be used to transform the value of a changing voltage to that of another magnitude, the system is commonly called a transformer. The high voltages required to make the spark plugs spark in a car engine are produced by a transformer also known as the ignition coil.

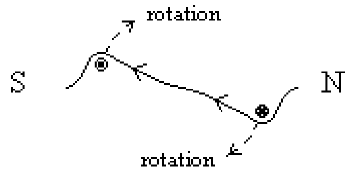
10.6 The Generator or Alternator

This form of induction is commonly used to generate electricity for all types of electricity supply, ranging from power generation for cities down to recharging car batteries as the engine runs. We will consider the effect of turning a rectangular coil in a uniform magnetic field.

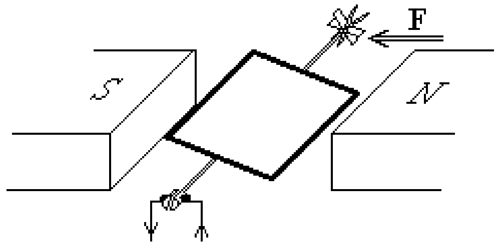


Our diagram shows a generator with a rectangular coil that rotates between two poles of a magnet, the coil is driven around a fixed axis by the force (**F**) driving a small fan or turbine. On the near side of the axis is a commutator, this acts as a sliding switch so that the generated current is can be drawn off.

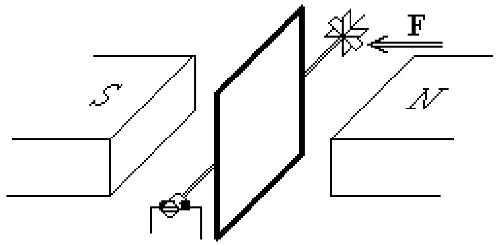
In our next diagram we see a cross section of the generator viewed as we look down the axis. This time the coil has begun to rotate.

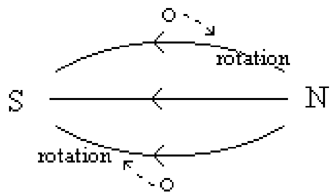


This cross section helps us determine the direction that the current flows through the commutator.



When the coil has turned so that its plane is vertical the induction arms of the coil are moving parallel along the field lines and no induction is occurring. The induced current is zero and the commutator is changing terminals so that the next surge of current will continue to flow in the same direction from the terminals.





By contrast an alternator does not need a commutator; instead of rotating a coil in a fixed magnetic field, the alternator has a fixed coil and a magnet is spun around inside the coil. A commutator is not needed as the terminal and connections remain stationary, however the alternating magnet poles give a current flow that runs in alternate directions, such a current is called an *alternating current* commonly abbreviated to a.c.