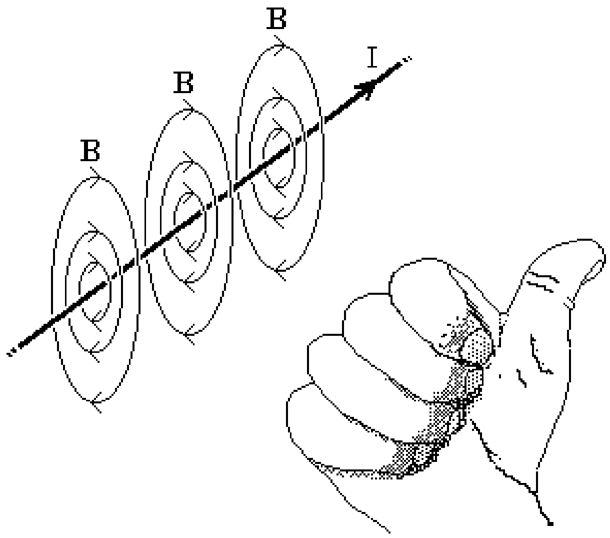
CHAPTER 9.

MAGNETIC FORCE

9.1 The magnetic field

A magnetic field surrounds all electric currents. Just as a current represents charges in motion, then the magnetic field can represent an electric field in motion. The shapes of magnetic field lines are, however quite unlike those of electric (properly electrostatic) field lines. The symbol for a magnetic field is most often **B** and the corresponding units of this field are *tesla,* T or *webers per square metre* Wb m^{-2}. The second definition draws our attention to the fact that the field is a type of density (with respect to an area), the webers are units of magnetic flux and the field vector **B** is more properly called the magnetic flux density, however many textbooks still call **B** the magnetic field.



In the above diagram a long straight wire carrying a current I, is shown with some of the magnetic field lines that surround the wire. The field lines do not indicate a direction of force. It is important to note that the magnetic field lines circulate about the wire with a *right handed* rotation; that is if the right thumb points in the direction that the current flows then the right fingers curl naturally in the direction that the field runs. The equation for the strength of the field a distance *r* from the wire, can be written as

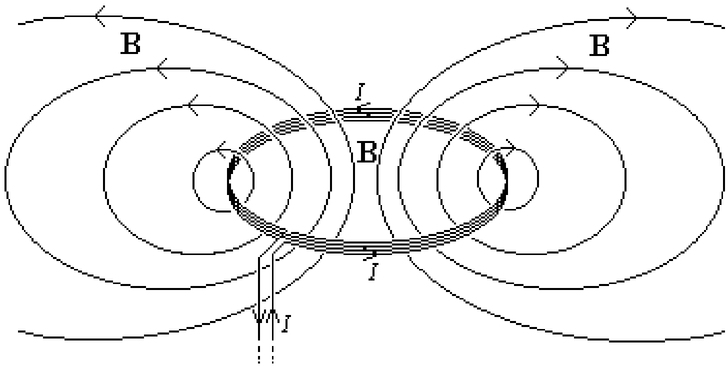
$$B = \mathrm{k_m} \frac{I}{r} \qquad (9.1)$$

where *I* is the current in the wire and k_m is a constant.

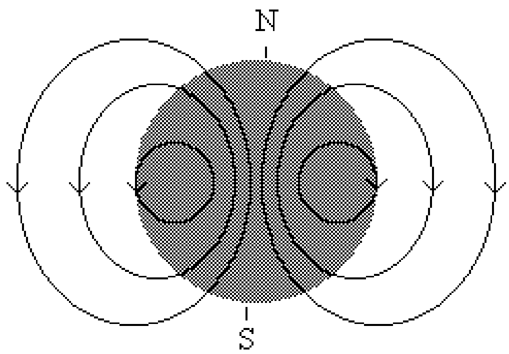
$$k_m = 2.00\times10^{-7} \quad \mathrm{T\,m\,A^{-1}}\ .$$

As the field lines form closed circles around a single conductor, *r* is also the radius of the constant field circle (contour of the field). It is useful to suppose that the field is perpendicular to the current that has caused the field. Magnetic field lines form closed contours they never meet or intersect (this happens for electric field lines at charges).

When a current flows around a closed loop a field is formed within and outside the loop. The concept of field lines suggests that they will be bunched inside the loop and spread out outside the loop so that a comparatively strong field will exist in the loop. As the lines form closed contours they will eventually return and link about the loop. The following diagram shows a cross section of field lines that link through a current carrying loop.

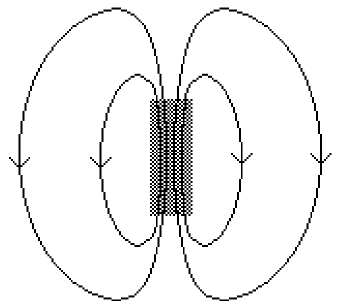


You should check that the field lines wrap around the current carrying conductors according to the right-hand rule.



It is well known that the Earth has a magnetic field, comparison of the above two diagrams suggests that this field is caused by charged particles circulating within the core of the earth. When comparing the magnetic field of a conducting circuit to that of a natural magnet, you should remember that the magnetic field lines run from south to north inside the magnet; this means that at the surface of the earth the outer field lines run from the north to the south pole.

Some materials are natural or permanent magnets. This is because the electrons inside the material run in microscopic aligned circuits. The overall effect is similar to that of the Earth but on a much smaller scale.



9.2 The Magnetic Force

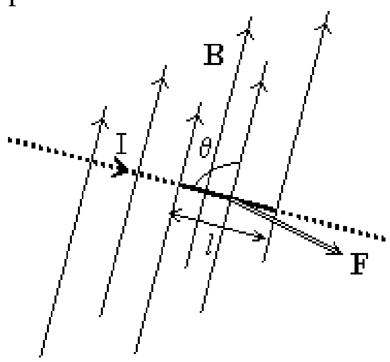
When a current flows in a magnetic field a force will occur between the field and the current carrying conductor. The force on the conductor will always be perpendicular to both the field and the current. The equation for the force an a length l of conductor carrying a current I is

$$F = BIl. \qquad (9.2a)$$

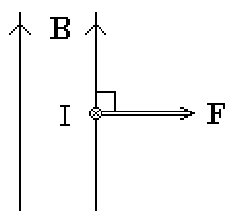
While the force must be perpendicular to the field and current, the field and current need not be perpendicular to each other. If the angle between the field and the current is θ then the magnetic force between the field and a length l of conducting wire carrying a current is

$$F = BIl\sin\theta \qquad (9.2b)$$

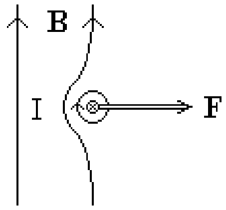
A diagram that helps illustrate this formula follows.



If we could view this situation by looking directly down along the current carrying conductor we might see



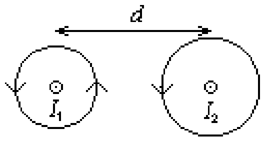
where the current moving away from the viewer has its motion shown as a cross (that represents the flights of an arrow moving away from the viewer). The force **F** is shown to be perpendicular to the field **B** as expected. A useful (but fanciful) way of considering the origin of the force **F** is to initially consider only the field created by the current I, the right hand circulation about the current will make this appear to run in a clockwise direction about the current. This field must then add to the applied field **B** so that, where it runs in the same direction the total field will strengthen. On the other side of the current where the fields are opposed the total field will be weaker.



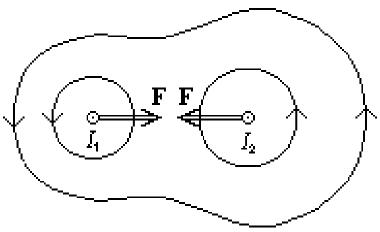
In our visual scheme we can suppose that the fields turn and bend as shown, the flow of the field lines remains consistent. The curvature of these lines then suggests that in order to straighten out and reduce the density of the field lines a force acts on the conductors consistent with the one we have defined.

9.3 The Force Between Two Conductors

In this section we assume that we have two conducting wires running parallel to each other a distance *d* apart. The diagram in the last section shows us a view down such a conductor with a representation of a current flowing away from the observer. If the current (or field lines) come towards the observer the representation is that of an arrowhead, a circle with a dot or point in the centre. Two such parallel wires are shown below, both have their currents flowing directly out of the page towards the observer.



As we draw more magnetic-field lines further away from the wires they start to overlap because they can't cross we must let them circulate together as shown in the next diagram.



The common circulation suggests that the wires are drawn together. The field at I_2 caused by I_1 is

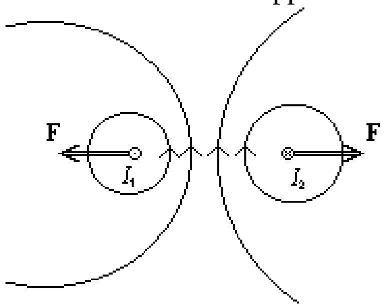
$$B = \mathrm{k}_{\mathrm{m}} \frac{I_1}{d}$$

and the strength of the force on a length *l* of I_2 caused by I_1 is therefore

$$F = k_m \frac{I_1 I_2}{d} l .$$

Forces of the same strength act to draw each wire together.

A second case arises when the currents flow in opposite directions.



In this case the field lines must collect between the two wires suggesting that they will be forced apart. The force on each wire for a length l remains

$$F = k_m \frac{I_1 I_2 l}{d}$$

9.4 The Force on a Moving Charge

From a previous section (**The Potential in a Resistor**) we have the equation (7.6) for the electric current

$$I = \frac{q - q_0}{t - t_0} = \frac{q}{t} .$$

Replacing this in equation (9.2a) will give us

$$F = B\frac{q}{t}l .$$

Now the time t is the time for any of the current charges to effectively move along the length l. These charges have an effective speed $v = l/t$, so that we expect a magnetic force of

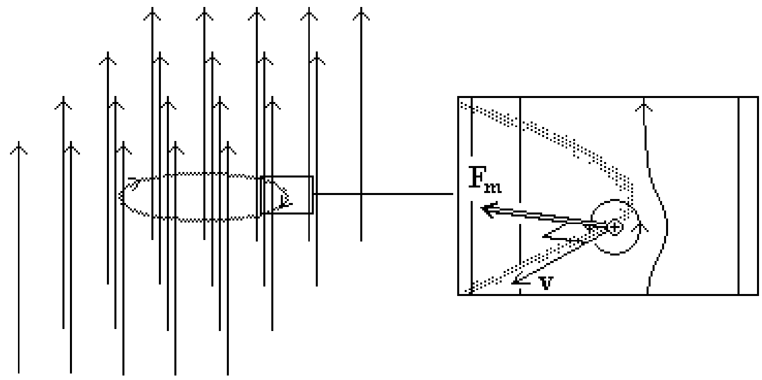
$$F = qvB \qquad (9.3a)$$

whenever a charge q moves through a magnetic field B with a speed v. The direction of the velocity of the charge should be the same as the direction in which the current flowed when we applied the previous equations (9.2a) and (9.2b). When the velocity of the charge makes an angle θ with the direction of the magnetic field our equation becomes

$$F = qvB\sin\theta \qquad (9.3b)$$

The force on the charge will always be perpendicular to both the field and the velocity of the charge.

A special case can be immediately considered, that is the case where a charge (or a number of charges) enter a magnetic field with a velocity **v** that is perpendicular to the field.



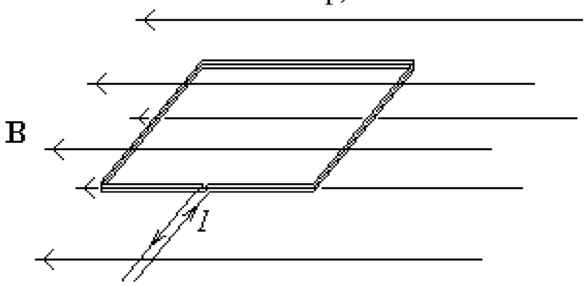
As shown in the smaller enlargement the charge entering the field with a velocity **v** has an associated magnetic field which enables us to visualise the formation of the magnetic force $\mathbf{F}_m$ which is perpendicular to **v**. This perpendicular force moves the charge (or charges of the same speed) around a circular path of radius *r*. The magnetic force pulls the particles from their tangential line and circular motion occurs when the reaction to being pulled from a straight line balances the magnetic force on each charge. If the mass of each charge is *m* we have

$$qvB = \frac{mv^2}{r}.$$

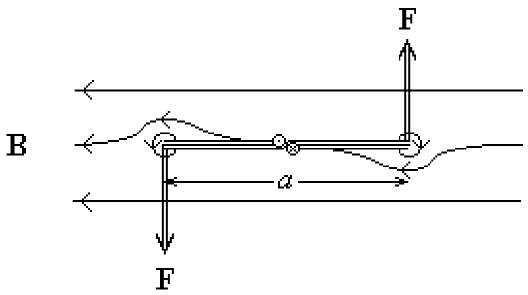
This equation becomes useful when studying modern physics.

9.5 The Forces on a Current Loop

For our purposes we will suppose we have a square loop with N turns and a current I that passes through each turn. This square with sides of length a is placed in a uniform magnetic field B that runs parallel across the face of the loop, as shown below.



If we view this system looking down the input current we will see it as shown in the next diagram.



There are two forces acting on the loop, they act in opposite directions and both have a strength of

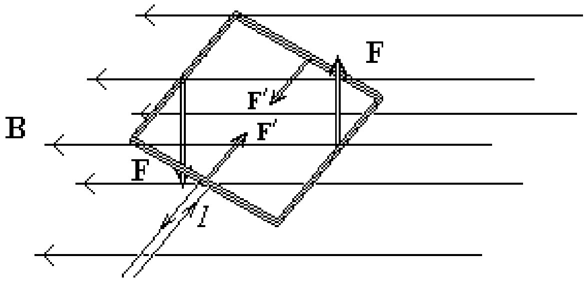
$$F = BNIa$$

Because these forces are equal and opposite the loop cannot accelerate as no total force acts on it. However the loop will turn about the central axis (where the current flows in or out). The loop has a turning moment (also called the torque) of

$$2NIa\frac{a}{2} = NIAB\,.$$

Where $A = a^2$ is the area of each loop. The other two sides of the loop experience no force at all because they run parallel to the field, when the current and field are parallel then the magnetic force is zero.

As the coil turns about the axis the moment changes. The two forces **F** keep the same magnitude but their lines of action come closer together, thus reducing the turning moment.



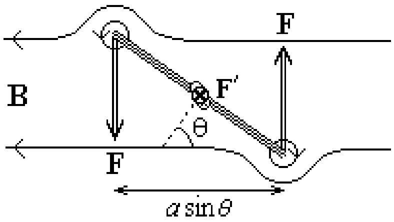
Although the two forces **F** remain the same in strength and direction, the separation of their lines of action reduces to $a\sin\theta$ (where the angle θ is shown in the next diagram) and hence their turning moment reduces to

$$\text{moment} = NIAB\sin\theta\,.$$

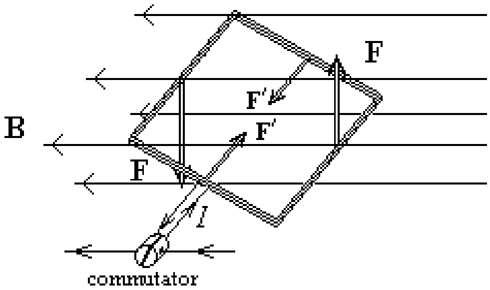
The sides that were parallel to the field **B** are no longer parallel, so new forces **F'** (also shown in the last diagram) appear as the coil rotates. These two new forces are

$$F' = BNIa\cos\theta$$

they always oppose each other directly and give neither acceleration or turning moment to the system.



To make a motor we apply the current to the coil through a rotary switch called a commutator, in this each lead from the coil is connected to one of two semicylindrical conductors that rub against brushes that supply the current. In this way when the turning moment reduces to zero, the inertia of the coil pulls it past the perpendicular and the switch changes the current direction through the coil, this causes the coil to continue to turn.



This type of direct current motor is similar to the type used as starters for car engines.