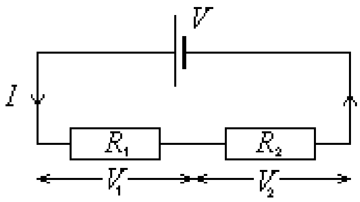
CHAPTER 8.

CIRCUITS

8.1 Series resistance

Two resistors arranged so that the current passes first through one, and then the other, are said to be *in series.*



For each resistor we have

$$V_1 = IR_1 \quad \text{and} \quad V_2 = IR_2$$

and by accounting for the energy used in the circuit we have

$$V = V_1 + V_2 = IR_1 + IR_2 \, .$$

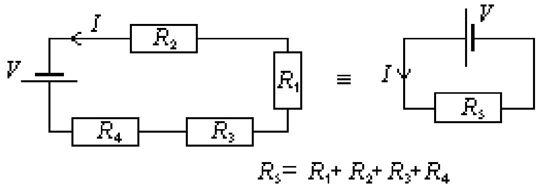
The effect of these two series resistors is to act as a single larger resistor called R_s. Just as

$$R_1 = \frac{V_1}{I} \quad \text{and} \quad R_2 = \frac{V_2}{I}$$

we can also have $$R_s = \frac{V}{I} = \frac{V_1}{I} + \frac{V_2}{I}$$

so that $$R_s = R_1 + R_2 \, . \qquad (8.1)$$

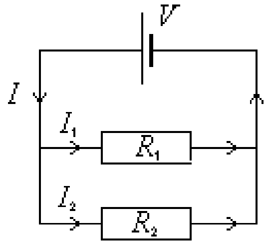
This calculation enables us to replace any two or more resistors in series, by a single resistor equal to the sum of the resistors found in series without changing the current in the circuit.



The two circuits shown above are equivalent.

8.2 Parallel resistance

Two resistors arranged so that a fraction of the current can pass through either one or the other, are said to be *in parallel*.



For each resistor we have

$$V = I_1 R_1 = I_2 R_2$$

and because of charge conservation we also have

$$I = I_1 + I_2 .$$

Now if we consider these two resistors to be equivalent to one parallel resistor R_p, then we can write

$$\frac{V}{R_p} = I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

or

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad (8.2)$$

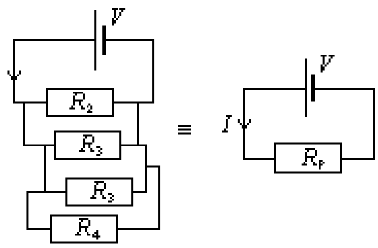
which can be also written as

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \quad . \qquad (8.3)$$

If we have three or more resistors all in parallel we can still use

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} .$$

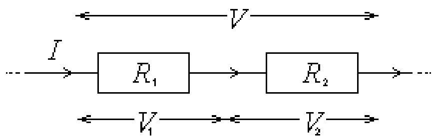
So that the four resistors in the following diagram act as a single parallel resistor R_p.



The two circuits shown above are equivalent.

8.3 Voltage division

When a current flows through two series resistors an intermediate voltage appears between the two resistors, thus the total voltage across both resistors can be broken down to provide a smaller voltage.



For the above circuit fragment

$$V = I(R_1 + R_2)$$

so that

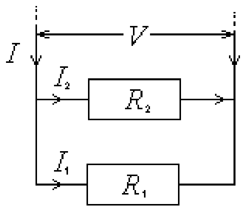
$$V_1 = IR_1 = \frac{R_1}{R_1 + R_2}V \qquad (8.4a)$$

and

$$V_2 = IR_2 = \frac{R_2}{R_1 + R_2}V \qquad (8.4b)$$

8.4 Current division

When a current flows through two parallel resistors, arranged side by side as in the circuit below, the current breaks into two smaller currents that flow through each of the two resistors, thus the total current in the circuit is broken into two smaller currents.



For the above circuit fragment

$$V = IR_p = \frac{R_1 R_2}{R_1 + R_2} I$$

so that

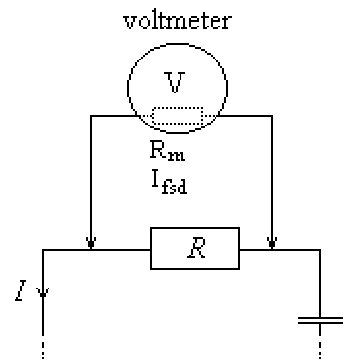
$$I_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I \qquad (8.5a)$$

and

$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I . \qquad (8.5b)$$

8.5 Voltmeters

A voltmeter is an instrument with two conducting probes that can be used to touch two points in a circuit and measure the voltage or potential difference between those points.

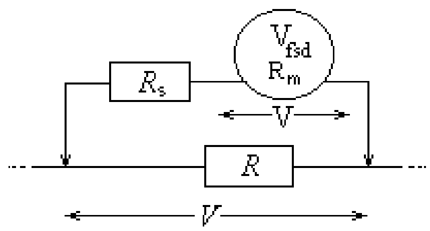


In the diagram a voltmeter V is used to measure the potential across a resistor R, the leads from the meter are shown touching the conducting wire on either side of the resistor which is part of a closed circuit, a current I flows through the resistor. The voltmeter itself has an internal resistance R_m and is also characterised by a maximum current I_{fsd} that can flow through the meter before it becomes damaged or over loaded. The letters "fsd"
stand for "full scale deflection" of the meter. The corresponding voltage that the meter will measure at full scale deflection is

$$V_{fsd} = I_{fsd} R_m .$$

The purpose of a voltmeter is to measure voltage, not current, for this reason voltmeters have high internal resistances R_m. A typical value would be about 10^6 Ω or 1 MΩ.

Sometimes the full scale deflection of a voltmeter is too small to allow the measurement of a larger voltage, this situation can be remedied by using the voltage division principle of equations (8.4).



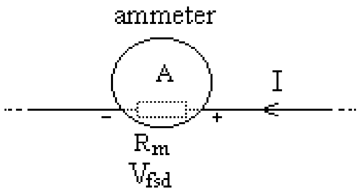
For the above diagram these equations give

$$\mathrm{V} = \frac{R_m}{R_m + R_s} V .$$

If $R_s + R_m$ draw current from the circuit then the voltmeter has disturbed the circuit that is being measured, this is the reason that $R_s + R_m >> R$. The circuit shown above enables one to use a voltmeter to measure higher voltages above the full scale range of the voltmeter. The extra resistor R_s is called a shunt resistor.

8.6 Ammeters

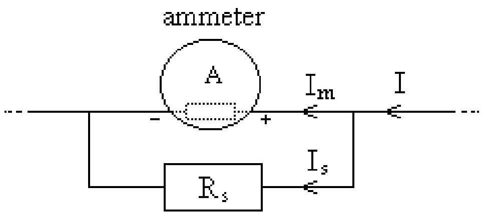
A amp-meter is abbreviated to ammeter and these are instruments must be inserted into a circuit to measure the current that flows in a circuit branch.



In the diagram an ammeter A is used to measure the current I flowing along a wire. The sign of each ammeter terminal are usually indicated on the ammeter, (red is + and black is -). The ammeter must have a very low internal resistance R_m so that it does not in itself impede the flow of current or develop a back e.m.f.. Again the meter is characterised by a maximum current I_{fsd} that can flow through the meter before it becomes damaged or over loaded.

$$I_{fsd} = \frac{V_{fsd}}{R_m}.$$

If we are required to measure a current greater than I_{fsd} we may do so using a parallel shunt resistor that enables the excess current to pass around rather than through the ammeter.



The situation in the above diagram is described by equations (8.5).

$$I_m = \frac{R_s}{R_m + R_s} I$$

and

$$I_s = \frac{R_m}{R_m + R_s} I.$$

If $R_s << R_m$ then the greater current will flow through R_s thus protecting the meter from overloading or burning out.