CHAPTER 7.

VOLTAGE AND CURRENT

7.1 Charge in a medium

As we now want to consider the movement of charges through space we will start by noting that we classify the surroundings of a charge in three ways: the first is as a vacuum, the second is as a conductor, while the third is as an insulator. While the idea of charge passing through a vacuum might sound rather abstract and scientific there is a common example found in the picture tubes of television sets. The material through which charges pass can also have the technical term of a *medium*. All media have the ability to allow the passage of charge in a manner somewhere between the extremes of an ideal conductor or an ideal insulator.

Now it is easy to describe the motion of a charge through a vacuum. This charge will only be influenced by an electric field, the acceleration of this charge will be

$$\mathbf{a} = \frac{q}{m}\mathbf{E}. \qquad (7.1)$$

where **E** is the field across the vacuum, and m and q are the mass and charge of the charge. Students are sometimes concerned by the absence of gravity when calculating charge motion; gravity is always present, but we assume that the electrical forces are so much greater so that we can ignore the gravitational effects.

In an perfectly insulating medium the charge will not be free to move or penetrate into the medium, it will be held stationary against the applied field that may act on it.

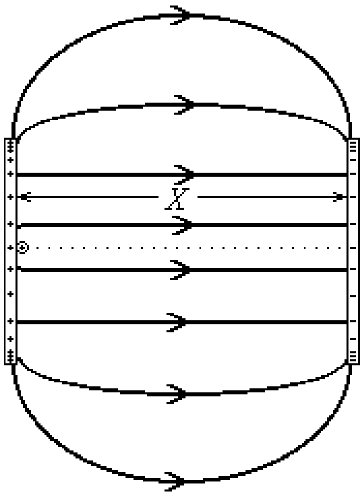
By contrast, in an ideal conductor the charge is perfectly free to move, however the conductor contains many other charges that are also equally free to move. When a charge tries to move in a conductor it immediately encounters other like charges (that mutually repel) and the charges exchange momentum. The initial momentum of this charge is transferred rapidly through the conductor (with a speed near that of light) and another identical charge on the other side of the conductor will carry the momentum out from the conductor. In a perfect conductor there is no internal momentum loss and the other free charges arrange themselves to cancel or prevent any accelerating field from penetrating into the conducting medium. This description remains consistent with the two important conservation rules, the conservation of momentum and the conservation of charge.

In a semi-conducting or *resistive* medium the momentum from the initial force $q\mathbf{E}$ is slowly reduced as it passes through the resistance. The field **E** must spread through the medium so that as momentum is lost to resistance, the field overcomes this loss. The momentum still passes from charge to charge, while the field in the medium ensures that the momentum loss is overcome by the local forces. As before final charge is leaves the resistance with the same kinetic energy as the initial charge, even though energy was lost across the resistive medium.

7.2 The uniform electric field in free space

In the section on **Motion with constant acceleration** we learned how to make calculations for the special case of constant acceleration. We can apply these concepts to the motion of a charge provided the acceleration is constant, this will be the case in a vacuum. In a resistance the average acceleration is zero as the field balances the resistive loss, all the same zero acceleration is constant acceleration.

A constant field is found between and near the centres of two separate parallel conducting plates, one charged with positive charges and the other charged with the same amount of negative charge. This is illustrated in the following diagram of field-lines, whose directions are indicated by arrows.



The field **E** runs straight down the middle of the plates (we have included small kinks because of the effect of the extra charge that has been placed in the field). If a small charged particle of charge $+q$ and mass m, is released at the surface of the positive plate in the centre, it will accelerate along the dotted path that is shown, this path has the same direction as the nearby field lines and the field is constant along this path. The length of this path is X and the particle accelerates with a constant acceleration given in equation (7.1). The equation for motion with constant acceleration gives

$$v_-^2 = v_+^2 + 2aX = v_+^2 + 2\frac{qE}{m}X$$

where v_+ and v_- are the speeds at the +ive and -ive plates. From this equation we can find the increase in kinetic energy as the particle accelerates between the plates,

$$\tfrac{1}{2}mv_-^2 - \tfrac{1}{2}mv_+^2 = qEX \quad . \qquad (7.2)$$

Now the increase in kinetic energy is equal to the term qEX, this quantity must have the matching units of Joules. The increase in the energy of the particle has come from the electric field, potential energy has been reduced as kinetic energy was gained. To account for this potential energy change, there is an important term called the electric *potential*. We may also use the term *voltage*, or when chemical energy is available to send a charge around a circuit we use the term *electro-motive force* abbreviated as e. m. f.. The electric potential is ***the***

potential energy per charge that is available when a charge is placed in field. This definition matches that of the electric field; that is the force per charge at a point in a medium.

All energy is measured by observing an energy change and this must be the case with electric potential, if we use V as our symbol for potential we can write the last equation as

$$\tfrac{1}{2}mv_-^2 - \tfrac{1}{2}mv_+^2 = qEX = q(V_+ - V_-) \qquad (7.3)$$

where V_+ is the potential at the positive plate and V_- is the potential at the negative plate. From this particular equation we can also relate the potential difference or voltage difference to the electric field as

$$E = -\frac{V_- - V_+}{X} \qquad (7.4)$$

where we have used a formula to say that the electric field is the negative gradient of the electric potential.

The formula given in equation (7.3) can be applied to circumstances where the field and therefore the acceleration, of a free charge is not constant, the equation can be written with no reference to the field E or the separation of the plates X as

$$\tfrac{1}{2}mv_-^2 - \tfrac{1}{2}mv_+^2 = q(V_+ - V_-). \qquad (7.5)$$

7.3 The potential in a resistor

When a charge crosses a vacuum under the influence of a field it accelerates all the time it is in the field. When a charge crosses a resistance under the influence of a field it constantly loses momentum to the resistance and regains this momentum from the field, in this case it moves with a steady average speed and no average acceleration. Instead of the above equation for kinetic energy, we write

$$\text{Energy lost to the resistance} = q(V_+ - V_-).$$

Now what happens if we increase the potential difference that drives the current through the resistance? Clearly more energy will be lost to the resistance and yet the charges do not gain more kinetic energy, what happens is that <u>more charges</u> are driven across the resistance as the voltage is increased. Now the amount of charge crossing the resistance also depends on the time over which the determination is made and so the effect of increasing the voltage is to increase the rate of flow of charge, this is also called the *electric current I*. A suitable equation for current is

$$I = \frac{q - q_0}{t - t_0} = \frac{q}{t} \qquad (7.6)$$

Here we have written the current as the amount of charge $q - q_0$ that flows through a resistor in time $t - t_0$, or more simply q in time t. The units for electric current are amperes (symbol A), the ampere is a base units for our modern system of units, however it is obviously equivalent to coulombs per second (C s^{-1}).

Returning to our energy considerations we have

$$V_- - V_+ \propto I$$

or by using V to represent the potential difference we have

$$V \propto I \quad .$$

For simple resistances we have Ohm's law

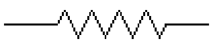
$$V = IR \qquad (7.7)$$

where the constant of proportionality R is a feature of the medium through which the charge passes, this property is called *resistance* and the measure of resistance is in units of ohms (symbol Ω).

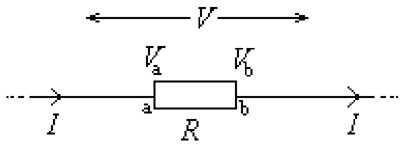
7.4 Circuit conventions

In writing equation (7.7) we have abbreviated the electric potential difference to what is simply called the voltage V. By convention the charge that flows in a current is assumed to be positive, therefore the charge must flow from a positive region, or potential to a negative region, or potential. In point of fact the most common form of charge flow is that of electrons from a negative potential to a positive one, but this fact does not change our calculations so we stay with the traditional convention of positive charges.

In the past the symbol for a resistor was

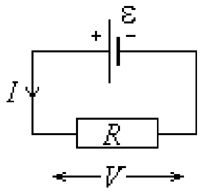


although a plain rectangle is now used. In the diagram below we have shown a section of a circuit that contains a resistor.



Because there is a potential difference V_b - V_a across the resistor, a current I flows through the resistor. Energy is lost in the resistor as charges move through the resistor, conservation of charge demands that the same amount of charge moves out of the resistor as that which moves in. The conventional charges enter the resistor at end **a** and lose energy in the resistor emerging with lower energy at end **b**. The energy per charge or potential V_a, is higher than V_b; $V = V_a - V_b$ is called the *back e. m. f.* as it acts against the potential that drives the charges around the circuit. As no energy is lost in the conducting wires there is no drop in potential along these wires, nor is there a field in them. For this reason the arrows indicating the voltage difference V in the diagram can end anywhere along the wire; the whole of the left hand section of the wire is at V_a while the whole of the right hand section is at V_b.

The symbol for a source of potential can be either a rectangle with a statement about the nature of the source, or the symbol shown in the following circuit diagram.



In this diagram we seen a cell (with a positive *anode* and a negative *cathode*). The voltage for this cell is ε which also stands for e. m. f.. For this, the most simple of circuits, we have

$$\varepsilon = V = IR\,.$$

The electric potential usually comes from a cell, a battery, a power supply or a generator.

7.5 Power Lost in a Circuit.

In section **7.2** we learned that "The electric potential is ***the potential energy per charge*** that is available when a charge is placed in field." We then used the single symbol V to represent the potential difference and went on to call this quantity the voltage. If V is the energy per charge we can write the energy gained by a charge in a circuit as

$$U = q\varepsilon$$

and the energy lost by a charge as it passes through a resistor (the voltage drop is also called a *back e.m.f*) is

$$U = qV\,.$$

Further the power loss is the rate of change of this energy loss, or to put it quite simply

$$P = \frac{qV}{t} = IV \qquad (7.8)$$