CHAPTER 6.

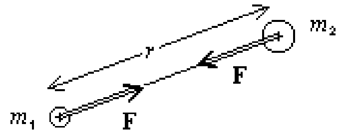
GRAVITY AND THE ELECTRIC FORCE

6.1 Gravity and electric force

The formulae for the gravitational forces between two masses and the electrostatic forces between two charges are similar, even though other physical details are quite different. For gravity the formula is

$$F = \mathrm{G}\frac{m_1 m_2}{r^2} \qquad (6.1)$$

where G is a constant, m_1 and m_2 are masses and r is the separation of the centres of mass of these two masses. The force **F**, acts with equal magnitude F on each mass and both masses are pulled together directly along the line between the two masses.



According to the formula (6.1) the strength F of the gravitational force increases as the two masses come together (it will decrease if they are pulled apart by some external system). Because the force is proportional to r^{-2} the formula is also called an "inverse square" law. The value for the constant is

$$\mathrm{G} = 6.673\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}.$$

For electrostatic attraction (or repulsion) the formula is

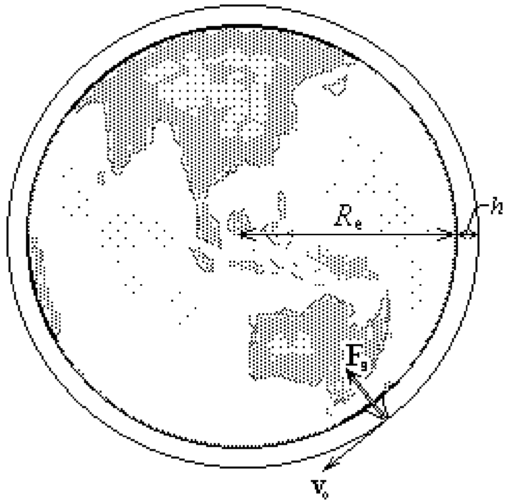
$$F = \mathrm{k_e}\frac{q_1 q_2}{r^2} \qquad (6.2)$$

where $\mathrm{k_e}$ is a constant, q_1 and q_2 are charges and r is the separation of these two charges. The concept of charge q is a new; charges have an extra property when compared with masses, they may be either +ive or -ive. An extra rule is required for the description pf the force between charges, *charges with the same sign repel while those with different signs are attracted*. The force **F**, acts with equal magnitude F on each charge and both charges are pulled together or repelled directly along the line between the two charges. The units of charge are coulombs, abbreviated as C. The value for the constant is

$$\mathrm{k_e} = 8.988\times10^{9}\ \mathrm{N\ m^2\ C^{-2}}.$$

6.2 Gravity

Although satellites made by people first appeared about the earth in 1958, the idea had been proposed by Isaac Newton. When Newton watched an apple fall in an orchard, he could possibly see the moon in the sky at the same time and he pondered why it was that apples fell to earth while the moon did not. Newton also speculated that an object thrown from a high mountain with sufficient horizontal velocity would fall over the horizon without reaching the earth's surface and could continue to do so until it returned to the mountain top from which it was launched. In a similar fashion the moon falls around our earth in a path called an orbit, each orbit takes about a month.



In the above picture the path of a satellite launched horizontally above New Zealand is shown passing around the earth. The initial horizontal velocity $\mathbf{v}_0$ is shown and the gravitational force between the satellite and the centre of mass of the earth $\mathbf{F}_g$ is also shown. While the circular path of the orbit is plausible it is also unusual; if the satellite was launched with a much higher initial speed v_0 it would swing away into space along a hyperbolic path or orbit the earth in an ellipse, if the speed was much lower the satellite would crash to earth. The circular path shown is a special case.

There is only one force acting on the satellite and that is the gravitational attraction of the earth $\mathbf{F}_g$, this force overcomes the inertia of the satellite as it tries to travel out into space with straight line momentum. The force pulls the satellite around in a circle, the speed of the satellite does not change, the gravitational force changes the direction of the velocity. The reaction to this force is the circular reaction given previously (equation 3.13), our force equation

$$\sum \mathbf{F} = m\mathbf{a}$$

becomes

$$G\frac{m_e m_s}{\left(R_e + h\right)^2} = \frac{m_s v_0^2}{R_e + h}. \tag{6.3}$$

In equation (6.3) m_e is the mass of the earth, m_s is the mass of the satellite, R_e is the radius of the earth and h is the height of the satellite above the earth's surface. Our original variable r has become the distance between the centres of mass of the earth and the satellite, $R_e + h$. The constant tangential speed of the satellite in the circular orbit can be written in terms of T the orbital time (or period) of the satellite and the distance around one orbit, this gives

$$v_0 = \frac{2\pi\left(R_e + h\right)}{T} \tag{6.4}$$

and combining equations (6.3) and (6.4) gives us

$$T^2 = \frac{4\pi^2\left(R_e + h\right)^3}{Gm_e}.$$

This equation enables us to find the period T of a satellite as it moves in a circular orbit at a given height above the surface of the earth. If we use our last equation we can also calculate the period of the early satellites, these had a height of about 100 km. We need to know that:

the mass of the earth $m_e = 5.98\times10^{24}$ kg

the radius of the earth $R_e = 6.37\times10^6$ m.
Then we can calculate

$$T^2 = \frac{4\pi^2 6470000^3}{6.67\times10^{-11}\times5.98\times10^{24}}$$

or

$$T = 5180 \text{ s} = 1.44 \text{ hr}.$$

The orbit of these satellites as recorded in the newspapers of the time was about 90 minutes.

The gravitational force on any object of mass m at the earth's surface is also called the weight of the object. According to the gravitational force equation (6.1) this is

$$W = G\frac{m_e m}{R_e^2}.$$

We have previously written the weight as

$$W = m\text{g}$$

where g = 9.8 m s^{-2}. This value can be confirmed using:

$$\text{g} = G\frac{m_e}{R_e^2} = \frac{6.67\times10^{-11}\times5.98\times10^{24}}{6370000^2} = 9.83 \text{ m s}^{-2}$$

and this is the acceleration caused by gravitational attraction to the earth.

6.3 Electron Orbits

The smallest atomic electric charge is associated with a tiny particle called the electron, while the dimensions of an electron can not be precisely defined the electron is known to have a mass, m_e of 9.11×10^{-31} kg and a charge, e of -1.60×10^{-19} C. As charge can be of two types, positive or negative, we might expect a corresponding small particle with a basic positive charge; this particle is the proton with a mass m_p of 1.67×10^{-27} kg and a charge of $+1.60\times10^{-19}$ C. Both the electron and proton are *quantum particles* and some other properties such as position, momentum and total energy are not precisely measurable.

As the two particles have charges of different sign they will be mutually attracted by the electric force defined in equation (6.2). Both particles will orbit each other (like the moon and the earth) but we will assume that the more massive proton remains in a fixed position while the lighter electron orbits about this proton. If the centres of mass of each particle are separated by a distance r then we can write an equation like (6.3) using instead the electric force of (6.2).

$$k_e\frac{e^2}{r^2} = \frac{m_e v_0^2}{r} \qquad (6.5)$$

Now the orbit radius can be estimated from the size of a typical atomic radius, about 2×10^{-10} m (see section **1.7 Estimations**). This information along with the other known constants give us a speed of

$$v_0 = \sqrt{\frac{k_e e^2}{m_e r}} = \sqrt{\frac{8.99\times10^9\times(1.6\times10^{-19})^2}{9.11\times10^{-31}\times2\times10^{-10}}} = 1.1\times10^6 \text{ m s}^{-1}$$

which is about right for the speed of an electron in orbit. We can also show that the period of the orbit is about

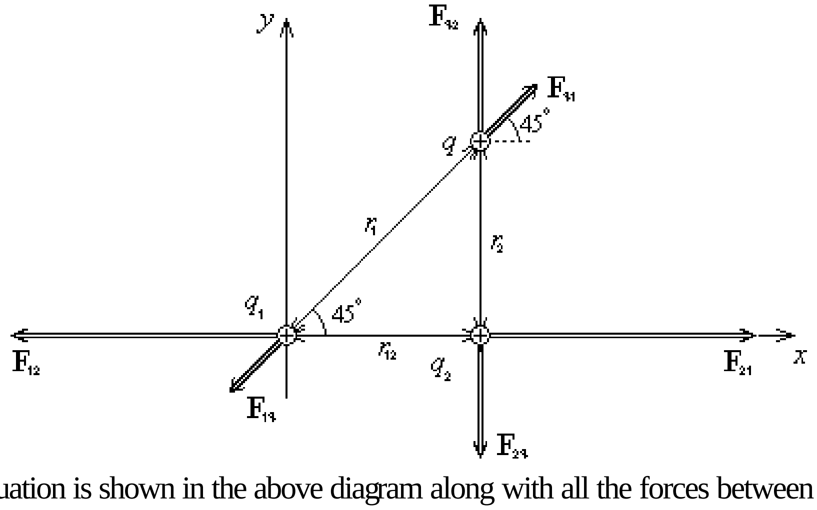
$$T = \frac{2\pi r}{v_0} = \frac{2\pi \times 2 \times 10^{-10}}{1.1 \times 10^{6}} = 1.1 \times 10^{-15} \text{ s}$$

This figure is of the same order as the period of the light waves that we can see with our eyes; this is hardly surprising as the light that we can see comes from changes of energy in electrons.

6.4 Three charges and electric forces

To help our understanding of the vector nature of forces we will consider the force on a charge q in the presence of two other charges q_1 and q_2. For the purposes of calculation we must assume that q_1 and q_2 have fixed positions and don't move. Ultimately all small particles vibrate with no fixed position but on a large scale we need to assume that they are fixed if our calculations are to proceed. If q, q_1 and q_2 all have the same sign then they will all mutually repel, the gravitational equivalent of mutual repulsion is not possible as the gravitational forces between masses are always attractive. Further with three or more charges it is not possible to devise an arrangement where all the charges are mutually attractive. This greater diversity with electric forces is the reason why we now consider electrical forces rather than the gravitational forces, otherwise our calculations follow identical procedures for either case.

Let us suppose that the three charges have values $q = 1.00$ C, $q_1 = 2.00$ C and $q_2 = 2.00$ C, and they are placed at the respective points (1.0, 1.0) m, (0,0) m and (1.0,0) m.



This situation is shown in the above diagram along with all the forces between charge pairs, these act along the lines of separation of the charges. As we only want to sum the forces acting on q our task is to sum $\mathbf{F}_{q1}$ (the force on q from q_1) and $\mathbf{F}_{q2}$ (the force on q from q_2). Using equation (6.2) we have

$$F_{q1} = k_e \frac{qq_1}{r_1^2} = \frac{2k}{\left(\sqrt{2}\right)^2} = k_e$$

and

$$F_{q2} = k_e \frac{qq_2}{r_2^2} = \frac{2k}{1} = 2k_e \ .$$

To find $\mathbf{F}_q = \mathbf{F}_{q2} + \mathbf{F}_{q1}$ we must write down and add the components of each force, for the x-components we have

$$F_{qx} = F_{q2x} + F_{q1x} = 0 + k_e \cos 45° = 0.707 k_e$$

and for the y-components we have

$$F_{qy} = F_{q2y} + F_{q1y} = 2k_e + k_e \sin 45° = 2.707k_e.$$

The total strength (or magnitude) of $\mathbf{F}_q$ is

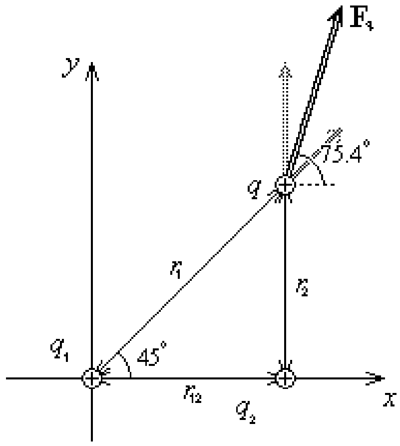
$$F_q = \sqrt{F_{qx}^2 + F_{qy}^2} = 2.80\, k_e$$

$$F_q = 25.2 \times 10^9 \text{ N}$$

(this is a very large force, most examples use micro-Coulombs for forces so that the results are in milli-Newtons). The angle the vector $\mathbf{F}_q$ makes with the x-axis is

$$\theta = \arctan\left(\frac{F_{qy}}{F_{qx}}\right) = \arctan\left(\frac{2.707}{0.707}\right) = 75.4°.$$

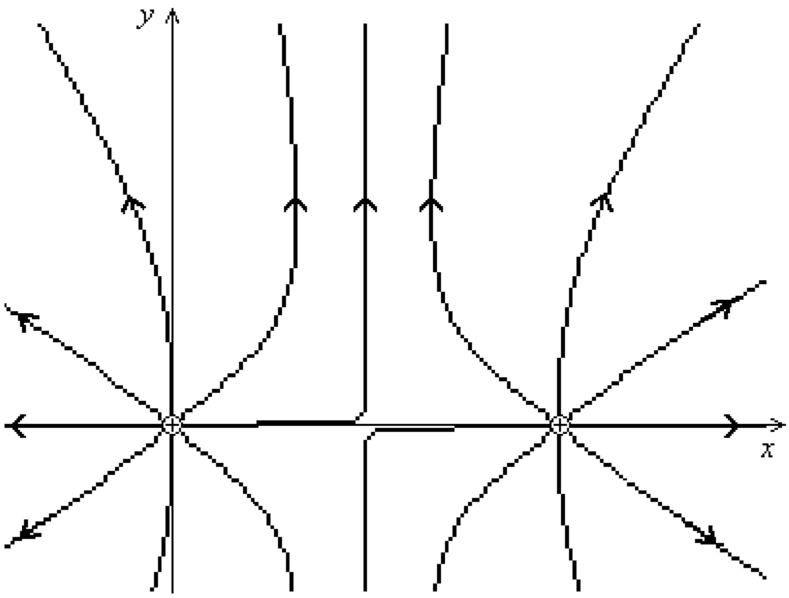
This resultant force is shown in the following diagram.



6.5 Field lines

In the last section we learned how to calculate the force from two charges q_1 and q_2 on to a third charge *q*. We should be able to repeat these procedures to find the force on *q* at any point (*x*,*y*) (except (0,0), where r_1 is zero and (1,0) where r_2 is zero). If we used a computer we could fill the diagram until it becomes black with vector arrows that showed the forces at each point. This diagram would not be very useful!

In order to picture what forces will act on a charge in the presence of other charges a scheme of *field lines* has been devised. To plot a field line we start at a point, calculate the force on a *test charge* (which is *q* in our case) and then move a small distance in the direction of that the force is pointing. When we then repeat this process in small steps (always moving in the direction of the last force) we will find that we have traced out a field line. These field lines show the direction, but not the strength (or magnitude) of the electric field at many points in space. Gravitational forces also give a Gravitational field. A copy of the result of this procedure is shown in the next diagram.



Now what we have done is use a test charge q to measure the influence of the two charges q_1 and q_2 throughout the surrounding space. This influence is detected as a force on the test charge at whatever point we place this test charge. The influence that we detect and measure is called a *field*, the strength of this field, E at any point is defined as the force per unit charge at that point. We can write an equation

$$E = \frac{F}{q} \tag{6.6}$$

where F is the strength of the force and q is the test charge that we have used to measure the field strength. As force is a vector then the field is also a vector and we can write

$$\mathbf{E} = \frac{\mathbf{F}}{q} \tag{6.7}$$

Field lines are a sophisticated and subtle subject; their properties are only mentioned here because they appear in more advanced syllabuses. The lines help indicate the direction of the forces that will found near charges. In some carefully prepared diagrams it is also possible to infer the relative strength of the forces by the density of these field lines. It should not be thought that a field line shows the path along which a free charge will accelerate, this is only true if a charge is released from rest on a straight field line and as a consequence accelerates along a straight line. If a charge tries to accelerate along a curved field line its inertial mass will naturally move it tangentially off this line.