CHAPTER 5.

FORCE, ENERGY AND POWER

5.1 Accelerated motion

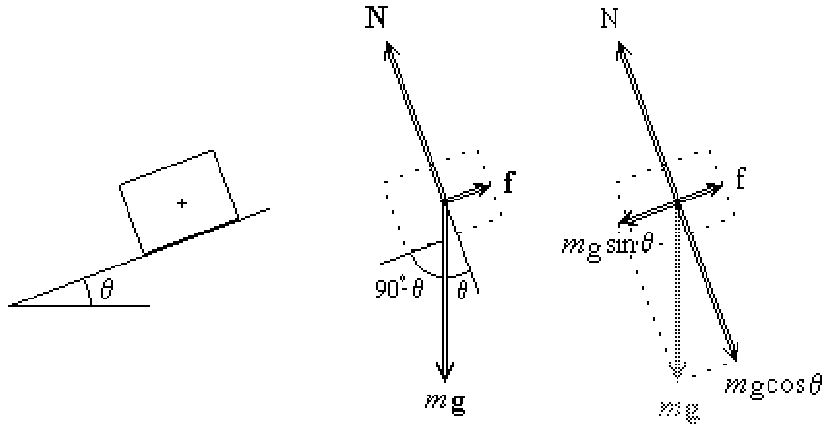
In a previous section we had equation (4.4) that gave the accelerated motion of a body, under the influence of unbalanced external forces, we could also call these non-equilibrium forces as they change the momentum of the body. We could also write our equation as

$$\sum_i \mathbf{F}_i = m\mathbf{a} . \qquad (5.1)$$

If the body is to accelerate without rotation, then all the forces $\mathbf{F}_i$ must meet at the centre of mass of the body or the sum of the moments of these forces must be zero.

5.2Analysis of acceleration caused by forces

A common textbook illustration of the application of equation (5.1) concerns a block that is able to slide down an inclined plane, such a system is shown in the next figure, along with the free body interpretation.



In the first part of the figure we see the block of mass *m* placed on a plane that has an incline angle of θ to the horizontal, we can also see a plane view of the centre of mass marked with a cross. In the second, middle diagram we have the *free-body* sketch showing the block and including all the forces that act on the block. These forces are:

(i) the weight $m\mathbf{g}$,

(ii) the normal force, **N** that acts at right angles to the inclined plane that supports the block,

(iii) the friction, **f** that acts between the block and the inclined plane and acts to hinder the sliding of the block down the plane.

In the last, figure on the rioght the force $m\mathbf{g}$ has been replaced by two mutually perpendicular components; it was necessary to erase or cross out $m\mathbf{g}$ so that it remains clear which forces are to be added up.

At this stage we are ready to write out equation (5.1) in terms of components, for the components parallel to the inclined surface we have

$$f - mg\sin\theta = ma_p \qquad (5.1a)$$

and for the components normal (that is perpendicular) to the inclined plane, we have

$$N - mg\cos\theta = ma_n. \qquad (5.1b)$$

For the first equation, (5.1a) we will work with the assumption that the block can only slide directly <u>down</u> the plane. For this to happen $mg\sin\theta$ must be greater than f so that

$$a_p = \frac{f}{m} - g\sin\theta$$

and a_p will be negative, this will make the block slide down the plane as expected. If a_p is zero then

$$f = mg\sin\theta$$

and the friction force up the plane cannot exceed the component of the weight force down the plane, this is a feature of frictional forces they are passive reactions that can reduce or prevent relative motion but cannot <u>create</u> relative motion. As the block is held by friction and cannot change momentum we have the sum of these force components as zero

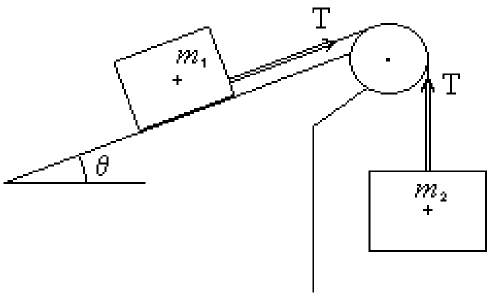
$$f - mg\sin\theta = 0$$

For the second equation, (5.1b) we cannot expect the block to jump into the air nor to force its way into the inclined plane, that is we must keep the condition $a_n = 0$, this being the case we have

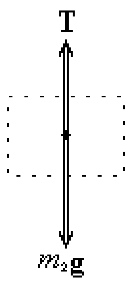
$$N = mg\cos\theta.$$

This shows us that the reaction to the component of the weight force into the plane is, as expected, equal to and in the opposite direction to this component of the weight force.

The problem we have just studied can be extended by joining two blocks together with a light unstretching cord that passes around a (mass-less) pulley as shown below.



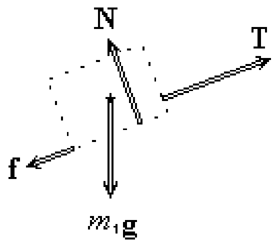
We have also shown that the blocks have masses m_1 and m_2, and that a tension T in the cord pulls m_1 up the inclined plane and supports m_2 against its weight force. It is not necessary for m_2 to be greater than m_1, for m_1 to move up the plane and we will assume that it will move this way. The next step in our analysis is to draw free-body diagrams that show all the forces acting on m_1 and m_2.

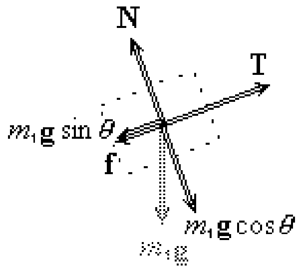


The free-body diagram for m_2 is straightforward, the centre of mass of must be in line with both the support force **T** and the weight force m_2**g**, if this were not the case the block would rotate to bring these forces into a line. Equation (5.1) can easily be written as

$$m_2 g - T = m_2 a \,. \qquad (5.1a)$$

The other free-body diagram for m_1 is more complicated, it shows the forces acting on m_1 drawn from where they would be expected to act.





As **f** and **T** are not in line they form a couple that would tend to rotate m_1 in a clockwise direction, however from the nature of the system we do not expect this rotation to occur, so we can conclude that the force averaged position of **N** has moved out of line with the centre of mass to counter this rotation. Once we have satisfied the condition that no rotation occurs, we can then draw all the forces acting from the centre of mass, in this case we have replaced m_1**g** with its components parallel to the inclined plane because this is the direction that motion will occur. We do not expect any normal motion directly into the inclined plane. Equation (5.1) can be now written as

$$T - m_1 g\sin\theta - f = m_1 a \qquad (5.1b)$$

and

$$m_1 g\cos\theta - N = 0 \,. \qquad (5.1c)$$

In writing these equations we have assumed that the block will accelerate up the plane as it is pulled by **T**, equation (5.1a) has the matching assumption that the force m_2**g** pulls the mass m_2 in a corresponding direction with the same acceleration so that the cord does not stretch.

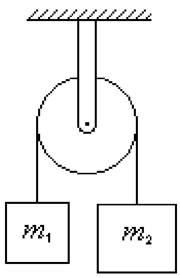
If we combine equations (5.1a) and (5.1b) we get

$$m_2 g - m_1 g\sin q - f = (m_1 + m_2)a \,. \qquad (5.1d)$$

The acceleration a of the blocks may be positive, that is m_1 accelerates up the plane, or negative, that is m_1 accelerates down the plane. The positive value occurs when m_2g is greater than $m_1$gsinθ + f in which case the weight force of m_2 is sufficient to meet our initial assumption.

This analysis can be applied to several other particular situations:

(i)



For this situation the equations become

$$m_2 g - T = m_2 a$$

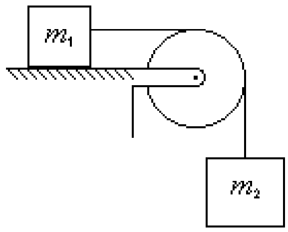
and

$$T - m_1 g = m_1 a$$

so that

$$a = \frac{(m_2 - m_1)g}{(m_2 + m_1)} \,.$$

(ii)



For this situation the equations become

$$m_2 g - T = m_2 a$$

and $T - f = m_1 a$

so that

$$a = \frac{m_2 g - f}{(m_2 + m_1)}.$$

In case (i) if our equations make a negative then m_2 goes up (rather than down as we initially assumed). In case (ii) some problems arise from the passive nature of the frictional resistance **f**, the value of f remains less than, or equal to m_2g and we cannot have m_2 being pulled upwards by **f** or m_1.

5.3 Motion and energy

In the previous chapter we introduced the concept of momentum by simply multiplying the velocity of an object by its mass. To define kinetic energy we take the same approach, we start with the equation for constant acceleration

$$v^2 = v_0^2 + 2a(s - s_0)$$

and divide this by one half then multiply both sides by the mass of the moving object, m to get

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = ma(s - s_0) = F(s - s_0). \qquad (5.2)$$

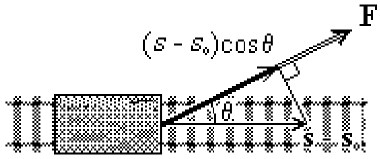
Here F is the magnitude of the outside force on the body that causes the constant acceleration, while $\frac{1}{2}mv^2$ is the kinetic energy of the body when it moves at speed v.

The units for energy are joules (symbol J) and you should note from equation (5.2) that

$$1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2} = 1 \text{ N m}.$$

Now kinetic energy is something that a body possesses, this energy is put into (or taken from) the body by work from the environment outside the body, this is the reason why we used the term external forces when discussing Newton,s laws. This work can be identified from the right hand term in equation (5.2); it is F(s - s_0). *The work done on a body is the product of the external force that acts on the body, times the distance that the force moves the body in the direction that the force is applied.*

"The distance that the force moves the body in the direction that the force is applied", needs an illustration. In the diagram below a wagon is pulled along a railway track by a force, **F** that acts to one side of the track. The wagon is constrained to move along the



track through a displacement **s - s**$_0$; although the wagon moves through this displacement the component of the vector in the direction of **F** is only (s -s_0)cosθ. The work done by **F** in moving the wagon through **s - s**$_0$ is given by

$$W = F(s - s_0)\cos\theta \qquad (5.3)$$

We can also use an equivalent statement *The work done on a body is the product of the distance that the force moves the body, times the component of the external force in the direction of this displacement.*

So far we have assumed that the force is constant in both magnitude and direction, we did this so that we could use a constant acceleration equation. In the situation shown above it could be possible that a friction force could oppose the action force **F**, so that little or no acceleration takes place and yet the force still does the work *W* against friction to move the wagon. If the waggon does not move at all, then (s - s_0) is zero and we are forced by our definition to say that **F** does no work.

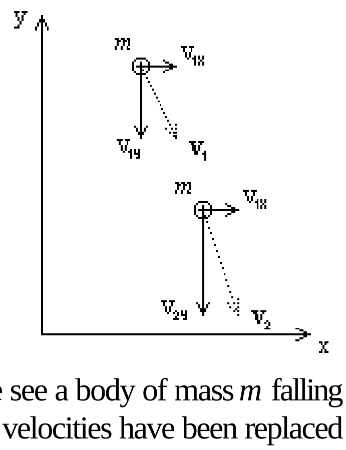
It is also likely that the force, is not constant in magnitude when this is the situation the work is calculated using calculus as

$$W = \int_{s_0}^{s} F\cos\theta \, ds \; .$$

As calculus is not required for this course we must replace this equation by the equivalent statement: *the work done by a force can be found from the area under the force against distance graph*. We could also say that the work is equal to the average force times the distance moved.

5.4 Kinetic and potential energy

In our familiar every-day environment we have become quite used to the effects of friction and other passive resistances, no matter what we do, or how we move we constantly lose work energy to friction. If these effects can be ignored (as may be the case with something falling through air with negligible air resistance) then equation (5.2) can be used to calculate the change in kinetic energy, this equation takes a form that is sometimes called the *Work Energy Theorem*.



In the diagram above we see a body of mass *m* falling freely, first with a velocity $\mathbf{v}_1$ then later with a velocity $\mathbf{v}_2$. These velocities have been replaced by their horizontal components v_{1x} and v_{2x} and their vertical components v_{1y} and v_{2y}. The horizontal component of the velocity cannot change with time as the only acceleration is vertically downwards. If the body has a velocity $\mathbf{v}_1$ at (x_1,y_1) and then falls to a new position (x_2,y_2) where it has a velocity $\mathbf{v}_2$, then we can write the equations for its motion as

$$v_{1x} = v_{2x}$$

and $$v_{2y}^2 = v_{1y}^2 - 2g(y_2 - y_1)$$

where $g = 9.8 \text{ m s}^{-2}$ is the downward acceleration due to gravitational attraction of the earth. By rearranging both equations and then multiplying by $\frac{1}{2}m$ we can arrive at

$$\tfrac{1}{2}m\left(v_{2x}^2 + v_{2y}^2\right) = \tfrac{1}{2}m\left(v_{1x}^2 + v_{1y}^2\right) - mg(y_2 - y_1)$$

or $$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv_1^2 - mg(y_2 - y_1) \qquad (5.4)$$

This equation can be written in a form that is both useful and demonstrates the conservation of kinetic plus potential energy at (x_1,y_1) and (x_2,y_2),

$$\tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_1^2 + mgy_1 \qquad (5.5)$$

where mgy is the gravitational potential energy of mass m at position (x,y). Although this is a workable definition of potential energy it is more correct to equate energy changes, in which case our Work Energy Theorem is best written as

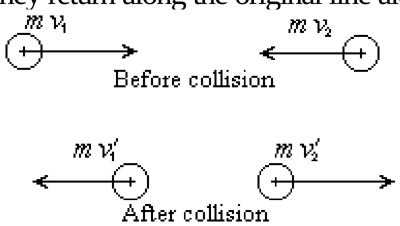
$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 = mg(y_2 - y_1).$$

Systems like this one, with no frictional loss of energy to the environment are called *conservative systems.* In this case $mg(y_2\text{-}y_1)$ is the work done by the gravitational force, this term indicates the potential for work to be done, hence it is called *potential energy.* As $\frac{1}{2}mv^2 + mgy$ is a constant the kinetic energy of a conservative system increases while the potential energy decreases and vice versa.

5.5 Kinetic energy and collisions

In a previous section we learned that: "two (or more) bodies are called a system and ... the momentum of a system cannot change unless a force from outside the system acts on the system". Momentum is always conserved for any instant when two bodies collide. This is not necessarily the case for kinetic energy. In an explosive collision or disintegration the kinetic energy of the fragments will increase while the total momentum must remain unchanged. In an *elastic* collision the kinetic energy will remain unchanged along with the momentum. For other collisions energy is lost to the environment, or to deformation of the bodies and the kinetic energy of the system reduces after the collision, even though the momentum is conserved during the instant of collision. These results do not easily fit our common experiences because we live in non-conservative systems where relative motion or momentum is constantly being lost to friction and other resistive effects.

There is a special interesting result that applies to elastic collisions where both momentum and kinetic energy are conserved. Consider two bodies of the same mass m, that approach each other and collide so that they return along the original line along which they travelled,



the two bodies are shown above with their velocity vectors and initial speeds v_1, v_2 and final speeds v_1', v_2'. The equation for the conservation of motion is

$$mv_1 - mv_2 = -mv'_1 + mv'_2$$

while the condition for the elastic collision requires constant kinetic energy or

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv'^2_1 + \tfrac{1}{2}mv'^2_2 .$$

There are two solutions for the speeds in these equations:

(i) $v'_1 = -v_1$ and $v'_2 = -v_2$

or (ii) $v'_1 = v_2$ and $v'_2 = v_1$.

In the first case (i) nothing has changed, perhaps the bodies never made contact? In the second case the bodies have exchanged momentum on collision. This result is important for the derivation of the ideal gas equation. At an atomic or microscopic scale collisions can usually be considered elastic and these systems will conserve their energy despite internal collisions.

5.6 Power

Power is the measure of the rate at which energy is used. For common physical examples this energy could be: heat energy, electrical energy, magnetic energy or mechanical work.
In the familiar case of a motor vehicle moving with a steady speed, we can say that the force from the engine just balances the friction and resistance forces and hence the vehicle is not accelerating. The work done by the engine is

$$W = \mathrm{F}\left(s - s_0\right)$$

as in equation (5.2), if the vehicle takes a time t to go from s_0 to s then we can calculate the power P as

$$P = \frac{W}{t} = \mathrm{F}\frac{s - s_0}{t} = \mathrm{F}v \qquad (5.6)$$

where v is the constant (or average) speed of the vehicle.

The units for power are watts, symbol W.

$$1\ \mathrm{W} = 1\ \mathrm{J\ s^{-1}} = 1\ \mathrm{N\ m\ s^{-1}} = 1\ \mathrm{kg\ m^2\ s^{-3}}.$$